

# Superstring with Extrinsic Curvature Action

*G.K.Savvidy*<sup>1</sup>

*National Research Center Demokritos,  
Ag. Paraskevi, GR-15310 Athens, Hellenic Republic*

## Abstract

We suggest supersymmetric extension of conformally invariant string theory which is exclusively based on extrinsic curvature action. At the classical level this is a tension-less string theory. The absence of conformal anomaly in quantum theory requires that the space-time should be 6-dimensional.

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<sup>1</sup>email: [savvidy@inp.demokritos.gr](mailto:savvidy@inp.demokritos.gr)

A string model which is exclusively based on the concept of extrinsic curvature was suggested in [1]. It describes random surfaces embedded in D-dimensional spacetime with the following action

$$S = m \cdot A = \frac{m}{\pi} \int d^2\zeta \sqrt{h} \sqrt{K_a^{ia} K_b^{ib}}, \quad (1)$$

where  $m$  has dimension of mass,  $h_{ab}$  is the induced metric and  $K_{ab}^i$  is the second fundamental form (extrinsic curvature).<sup>2</sup> The action (1) is proportional to the length of the surface  $A$ . The last property makes the theory very close to the Feynman path integral for point-like relativistic particle because when the surface degenerates into a single world line the action (1) becomes proportional to the length of the world line

$$S = m A_{xy} \rightarrow m \int_X^Y ds \quad (2)$$

and the functional integral over surfaces naturally transforms into the Feynman path integral for a point-like relativistic particle (see Figure 1). For a string which is stretched between quark-antiquark pair the action is equal to the perimeter of the Wilson loop  $S = m(R + T)$ , where  $R$  is space distance between quarks, therefore at the *classical level string tension is equal to zero*. In the recent articles [1, 4] the authors demonstrated that quantum fluctuations generate an area term in the effective action

$$S_{eff} = m^2 e^{-\frac{4\pi}{D-3}} \int d^2\zeta \sqrt{h} h^{ab} \partial_a X^\mu \partial_b X^\mu, \quad (3)$$

that is, dynamical string tension  $m^2 \exp(-\frac{4\pi}{D-3})$ .

Our aim now is to introduce fermions and to suggest supersymmetric extension of this model. First we shall consider basic relations in the bosonic case and the corresponding quantization rules of this highly non-linear and higher-derivative theory. We shall introduce Ramond fermions using standard two-dimensional world-sheet spinors. The main difference with the standard superstring theory is that supersymmetry transformation contains higher derivatives<sup>3</sup>. We shall demonstrate that the absence of conformal anomaly in quantum theory requires that the space-time should be 6-dimensional<sup>4</sup>,  $D_c = 6$ .

We shall represent the gonihedric action (1) in the form [1, 4]

$$S = \frac{m}{\pi} \int d^2\zeta \sqrt{h} \sqrt{(\Delta(h) X_\mu)^2}, \quad (4)$$

here  $h_{ab} = \partial_a X_\mu \partial_b X_\mu$  is the induced metric,  $\Delta(h) = 1/\sqrt{h} \partial_a \sqrt{h} h^{ab} \partial_b$  is Laplace operator and  $K_a^{ia} K_b^{ib} = (\Delta(h) X_\mu)^2$ . The second fundamental form  $K$  is defined through the relations:  $K_{ab}^i n_\mu^i = \partial_a \partial_b X_\mu - \Gamma_{ab}^c \partial_c X_\mu \equiv \nabla_a \partial_b X_\mu$ , where  $n_\mu^i$  are  $D - 2$  normals and  $a, b = 1, 2$ ;  $\mu = 0, 1, 2, \dots, D - 1$ ;  $i, j = 1, 2, \dots, D - 2$ .

Below we shall consider a model which has the same action (4), but now will be treated as a functional of two independent field variables  $X^\mu$  and  $h_{ab}$ , that is, we shall consider

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<sup>2</sup>In the above theory extrinsic curvature term *alone* should be considered as fundamental action of the theory. There is no *area* term in the action and it is not quadratic in extrinsic curvature as it was in previous studies [2, 3].

<sup>3</sup>The supersymmetric extension of the model which has the action quadratic in extrinsic curvature form was considered in [5].

<sup>4</sup>This may be interesting for little string theory in 6-dimensions[7].

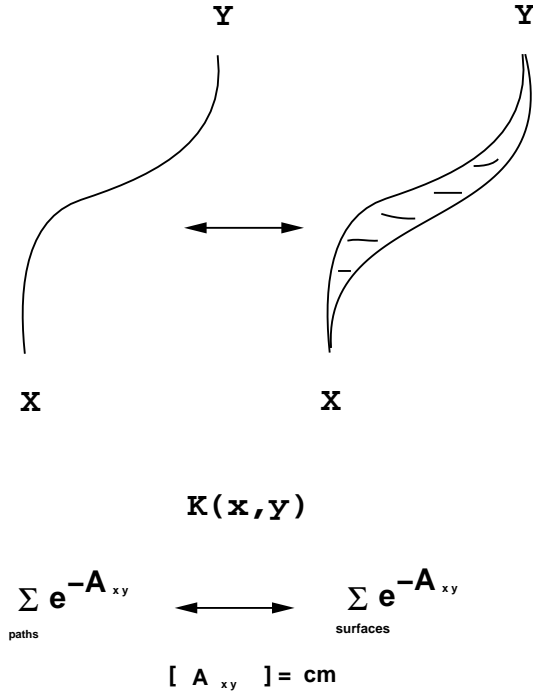


Figure 1: It is required that the action  $A_{xy}$  should measure the surfaces in terms of length, as it was for the path integral. When a surface degenerates into a single world line we have natural transition to a path integral .

scalar fields  $X^\mu$  in two-dimensional quantum gravity background  $h_{ab}$ . First we shall get equation which follows from the variation of the action over coordinates  $X^\mu$

$$\frac{\pi}{\sqrt{h}} \frac{\delta S}{\delta X^\mu} = \Delta(h) \left( m \frac{\Delta(h) X^\mu}{\sqrt{(\Delta(h) X)^2}} \right) = 0. \quad (5)$$

and shall introduce the momentum operator

$$P_a^\mu = \partial_a \Pi^\mu, \quad \Pi^\mu = m \frac{\Delta(h) X^\mu}{\sqrt{(\Delta(h) X)^2}},$$

where  $\Pi^\mu$  has the property very similar with the constraint equation for the point-like relativistic particle

$$\Pi^\mu \Pi_\mu = m^2. \quad (6)$$

One should also compute the variation of the action with respect to the metric  $h^{ab}$

$$\delta S = -\frac{1}{2\pi} \int \sqrt{h} T_{ab} \delta h^{ab} d^2 \zeta = 0 \quad (7)$$

with the following result for  $T_{ab}$

$$T_{ab} = \nabla_{\{a} \Pi^\mu \nabla_{b\}} X^\mu - h_{ab} h^{cd} \nabla_c \Pi^\mu \nabla_d X^\mu = 0, \quad (8)$$

where  $\{a \ b\}$  denotes a symmetric sum. The energy-momentum tensor is traceless  $h^{ab} T_{ab} = 0$  and we have interaction with conformally invariant matter field  $X^\mu$ . Thus our basic equations are (5),(6) and (8).

We can fix the conformal gauge  $h_{ab} = \rho \eta_{ab}$  using reparametrization invariance of the action and to derive it in the form (see (4))

$$\dot{S} = \frac{m}{\pi} \int d^2 \zeta \sqrt{(\partial^2 X)^2} = \frac{1}{\pi} \int d^2 \zeta \Pi^\mu \partial^2 X^\mu. \quad (9)$$

In this gauge the equation of motion (5) should be accompanied by the constraint equations (8) where now

$$\Pi^\mu = m \frac{\partial^2 X^\mu}{\sqrt{(\partial^2 X)^2}}.$$

In the light cone coordinates  $\zeta^\pm = \zeta^0 \pm \zeta^1$ ,  $\partial_\pm = \frac{1}{2}(\partial_0 \pm \partial_1)$  the constraints (8) take the form

$$\begin{aligned} T_{++} &= \frac{1}{2} (T_{00} + T_{01}) = \frac{1}{2} (\partial_0 \Pi^\mu + \partial_1 \Pi^\mu) (\partial_0 X^\mu + \partial_1 X^\mu) = 2 \partial_+ \Pi^\mu \partial_+ X^\mu, \\ T_{--} &= \frac{1}{2} (T_{00} - T_{01}) = \frac{1}{2} (\partial_0 \Pi^\mu - \partial_1 \Pi^\mu) (\partial_0 X^\mu - \partial_1 X^\mu) = 2 \partial_- \Pi^\mu \partial_- X^\mu, \end{aligned} \quad (10)$$

with the trace equal to zero:  $T_{+-} = 0$ . The conservation of the energy momentum tensor takes the form  $\partial_- T_{++} = \partial_+ T_{--} = 0$  and requires that its components are analytic  $T_{++} = T_{++}(\zeta^+)$  and anti-analytic  $T_{--} = T_{--}(\zeta^-)$  functions. Thus our system has infinite number of conserved charges. This residual symmetry can be easily seen in gauge fixed action (9) written in light cone coordinates  $\dot{S} = \frac{4m}{\pi} \int \sqrt{(\partial_+ \partial_- X^\mu)^2} d\zeta^+ d\zeta^-$ , it is invariant under the transformations  $\zeta^+ = f(\tilde{\zeta}^+)$ ,  $\zeta^- = g(\tilde{\zeta}^-)$  where  $f$  and  $g$  are arbitrary functions. Another important symmetry of the equation (5) is the transformation  $\partial^2 X^\mu \rightarrow \Omega(\zeta^+, \zeta^-) \partial^2 X^\mu$ , where  $\Omega(\zeta^+, \zeta^-)$  is an arbitrary function. Indeed if  $\partial^2 X^\mu$  is a solution of the equation (5), then  $\partial^2 \Phi^\mu = \Omega(\zeta^+, \zeta^-) \partial^2 X^\mu$  is also a solution.

Let us also consider additional conserved currents. If  $\delta X^\mu$  is symmetry transformation of the action with only second derivatives of the fields, then the corresponding conserved current is :

$$J_a = \partial_a \left( \frac{\partial L}{\partial (\partial^2 X^\mu)} \right) \delta X^\mu - \frac{\partial L}{\partial (\partial^2 X^\mu)} \partial_a \delta X^\mu.$$

In our case it is equal to:  $J_a = \partial_a \Pi_\mu \cdot \delta X^\mu - \Pi_\mu \cdot \partial_a \delta X^\mu$ . The action (9) is invariant under the global symmetries  $\delta X^\mu = \Lambda^{\mu\nu} X_\nu + a^\mu$ , where  $\Lambda^{\mu\nu}$  is a constant antisymmetric matrix while  $a^\mu$  is a constant. The translation invariance of the action (9)  $\delta_a X^\mu = a^\mu$  results into the conserved momentum current (already appearing in (5))

$$P_a^\mu = \partial_a \Pi^\mu, \quad \partial^a P_a^\mu = 0, \quad P^\mu = \int P_0^\mu d\zeta^1 \quad (11)$$

and Lorentz transformation  $\delta_\Lambda X^\mu = \Lambda^{\mu\nu} X_\nu$  into angular momentum current

$$M_a^{\mu\nu} = X^\mu \partial_a \Pi^\nu - X^\nu \partial_a \Pi^\mu + \Pi^\mu \partial_a X^\nu - \Pi^\nu \partial_a X^\mu, \quad \partial^a M_a^{\mu\nu} = 0, \quad M^{\mu\nu} = \int M_0^{\mu\nu} d\zeta^1 \quad (12)$$

In this set up the conformal transformation  $\zeta^+ = f(\tilde{\zeta}^+)$ ,  $\zeta^- = g(\tilde{\zeta}^-)$  is defined as  $\delta_c X^\mu = u^a \partial_a X^\mu$ , where  $u^0 = f + g$ ,  $u^1 = f - g$  and the corresponding conserved current is

$T_{ab}$  (8). We can now summarize the basic equations, constraints and boundary conditions for closed string in the form:

$$\begin{aligned}\partial^2 \Pi^\mu &= 0, & \Pi^\mu &= m \frac{\partial^2 X^\mu}{\sqrt{(\partial^2 X_\mu)^2}}, & \Pi_\mu^2 &= m^2, \\ T_{++} &= \partial_+ \Pi^\mu \partial_+ X_\mu = 0, \\ T_{--} &= \partial_- \Pi^\mu \partial_- X_\mu = 0, \\ X^\mu(\zeta^0, \zeta^1) &= X^\mu(\zeta^0, \zeta^1 + 2\pi).\end{aligned}\tag{13}$$

The general solution of the equation (5) can be represented in the form [6]

$$\Pi^\mu = \frac{1}{2} \Pi_R^\mu + \frac{1}{2} \Pi_L^\mu, \quad X^\mu = \frac{1}{2} X_R^\mu + \frac{1}{2} X_L^\mu + \frac{1}{2} \int [\Pi_L^\mu + \Pi_R^\mu] \Omega d\zeta^+ d\zeta^-,$$

where  $\Omega(\zeta^+, \zeta^-)$  is arbitrary function of  $\zeta^+$  and  $\zeta^-$ . From its definition the momentum density,  $P^\mu(\zeta^0, \zeta^1) \equiv P_0^\mu(\zeta^0, \zeta^1) = \partial_0 \Pi^\mu$  is conjugate to  $X^\mu(\zeta^0, \zeta^1)$ , therefore  $[X^\mu(\zeta^0, \zeta^1), P^\nu(\zeta^0, \zeta^1)] = i\eta^{\mu\nu} \delta(\zeta^1 - \zeta'^1)$  and one can deduce that the following commutation relations should hold

$$\begin{aligned}[\partial_+ X_L^\mu(\zeta), \partial_+ \Pi_L^\nu(\zeta')] &= i\pi \eta^{\mu\nu} \delta'(\zeta - \zeta'), \\ [\partial_- X_R^\mu(\zeta), \partial_- \Pi_R^\nu(\zeta')] &= i\pi \eta^{\mu\nu} \delta'(\zeta - \zeta'),\end{aligned}\tag{14}$$

with all others equal to zero  $[\partial_\pm X_L^\mu(\zeta), \partial_\pm X_L^\nu(\zeta')] = 0$ ,  $[\partial_\pm \Pi_L^\mu(\zeta), \partial_\pm \Pi_L^\nu(\zeta')] = 0$ <sup>5</sup>. To make these formulas more transparent to the reader let me use the analogy with the ghosts  $c^\pm$  and anti-ghost  $b_{\pm\pm}$  fields (or super-ghosts). The standard Faddeev-Popov action has the form  $\int c \partial_\pm b$  with nonzero anti-commutator only between  $c$  and  $b$  fields. Making use of these commutators one can get the algebra of constraints

$$\begin{aligned}[T_{++}(\zeta), T_{++}(\zeta')] &= i\pi(T_{++}(\zeta) + T_{++}(\zeta')) \delta'(\zeta - \zeta'), \\ [T_{++}(\zeta), P_+^2(\zeta')] &= i\pi P_+^2(\zeta) \delta'(\zeta - \zeta'), \quad [P_+^2(\zeta), P_+^2(\zeta')] = 0,\end{aligned}\tag{15}$$

with similar relations for  $T_{--}$  and  $P_-$ . Here  $2P_0^\mu = \partial_+ \Pi_L^\mu + \partial_- \Pi_R^\mu = P_+^\mu + P_-^\mu$ .

Our aim now is to include fermions. We shall introduce fermions into string theory with extrinsic curvature action using standard two-dimensional world-sheet Majorana spinors [8, 9, 10, 11, 12]

$$\Psi_A^\mu(\zeta) \equiv \begin{pmatrix} \Psi_-^\mu(\zeta) \\ \Psi_+^\mu(\zeta) \end{pmatrix},\tag{16}$$

where  $\mu$  is a space time vector index,  $A = 1, 2$  is a two-dimensional spinor index. The action is a sum of the bosonic and fermionic terms

$$\dot{S} = \frac{m}{\pi} \int d^2 \zeta \{ \sqrt{(\partial^2 X^\mu)^2} + i \bar{\Psi}^\mu \rho^a \partial_a \Psi^\mu \}\tag{17}$$

where  $\bar{\Psi}^\mu = \Psi^{+\mu} \rho^0 = \Psi^{T\mu} \rho^0$  and  $\rho^\alpha$  are two-dimensional Dirac matrices

$$\{\rho^a, \rho^b\} = -2\eta^{ab}.\tag{18}$$

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<sup>5</sup>Requiring that commutation relations for the energy-momentum tensor  $T_{++}$  and  $T_{--}$  should form the algebra of the two-dimensional conformal group one can get the same form of basic commutators.

In Majorana basis the  $\rho'$ s are give by

$$\rho^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \rho^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \quad (19)$$

and  $i\rho^\alpha\partial_\alpha$  is a real operator. The two-dimensional chiral fields are defined as  $\rho^3\Psi_\pm^\mu = \mp\Psi_\pm^\mu$ , where  $\rho^3 = \rho^0\rho^1$ , and our field equations can be written in a compact form

$$\partial_+\partial_-\Pi_R^\mu = 0 \Leftrightarrow \partial_+\Psi_-^\mu = 0, \quad (20)$$

$$\partial_-\partial_+\Pi_L^\mu = 0 \Leftrightarrow \partial_-\Psi_+^\mu = 0. \quad (21)$$

The symmetry transformation is:

$$\begin{aligned} \delta X^\mu &= \bar{\epsilon}\Psi^\mu, \\ \delta\Psi^\mu &= -i\rho^a\partial_a\Pi^\mu \epsilon, \end{aligned} \quad (22)$$

where the anti-commuting parameter  $\epsilon$  is a two-dimensional spinor

$$\epsilon \equiv \begin{pmatrix} \epsilon_- \\ \epsilon_+ \end{pmatrix}. \quad (23)$$

and  $P_a^\mu = \partial_a\Pi^\mu$  is the momentum operator. This transformation does leave the set of field equations

$$\partial^2 \Pi^\mu = 0, \quad \rho^a\partial_a\Psi^\mu = 0 \quad (24)$$

intact. Indeed:

$$\rho^a\partial_a\delta\Psi^\mu = \rho^a\partial_a(-i\rho^b\partial_b\Pi^\mu \cdot \epsilon) = -i\rho^a\rho^b\partial_a\partial_b\Pi^\mu \cdot \epsilon = i\partial^2\Pi^\mu \cdot \epsilon = 0 \quad (25)$$

$$\partial^2\delta\Pi^\mu = \partial^2\left\{ \frac{\eta^{\mu\nu} - \Pi^\mu\Pi^\nu}{\sqrt{(\partial^2 X)^2}} \bar{\epsilon} \partial^2\Psi^\nu \right\} = 0 \quad (26)$$

and field equations are invariant under transformation (22).

We should now test whether we have closed supersymmetry algebra of transformations (22). The commutator of two supersymmetries on  $X^\mu$  is given by

$$[\delta_1, \delta_2]X^\mu = \delta_1\bar{\epsilon}_2\Psi^\mu - \delta_2\bar{\epsilon}_1\Psi^\mu = \bar{\epsilon}_2(-i\rho^a\partial_a\Pi^\mu \epsilon_1) - \bar{\epsilon}_1(-i\rho^a\partial_a\Pi^\mu \epsilon_2) = 2i\bar{\epsilon}_1\rho^a\epsilon_2 \partial_a\Pi^\mu, \quad (27)$$

where the relation  $\bar{\epsilon}_2\rho^a\epsilon_1 = -\bar{\epsilon}_1\rho^a\epsilon_2$  has been used. The commutator of two supersymmetries on  $\Psi^\mu$  is given by

$$\begin{aligned} [\delta_1, \delta_2]\Psi^\mu &= \delta_1(-i\rho^a\partial_a\Pi^\mu \epsilon_2) - \delta_2(-i\rho^a\partial_a\Pi^\mu \epsilon_1) = \\ &= -i\rho^a\partial_a\left\{ \frac{\eta^{\mu\nu} - \Pi^\mu\Pi^\nu}{\sqrt{(\partial^2 X)^2}} \bar{\epsilon}_1 \partial^2\Psi^\nu \right\} \epsilon_2 \Leftrightarrow (1 \leftrightarrow 2) = \\ &= 2i\bar{\epsilon}_1\rho^b\epsilon_2 \partial_b\left\{ \frac{\eta^{\mu\nu} - \Pi^\mu\Pi^\nu}{\sqrt{(\partial^2 X)^2}} \partial^2\Psi^\nu \right\} + i\bar{\epsilon}_1\rho^b\epsilon_2 \rho_b \rho^a\partial_a\left\{ \frac{\eta^{\mu\nu} - \Pi^\mu\Pi^\nu}{\sqrt{(\partial^2 X)^2}} \partial^2\Psi^\nu \right\} = 0. \end{aligned} \quad (28)$$

Last two terms are equal to zero on mass shell because of the classical equation  $\partial^2\Psi^\mu = 0$ . The above calculation makes use of the Fierz rearrangement as well as the properties of Majorana spinors.

It is easy to prove that the action is invariant under the transformation of equation (22). The invariance is achieved without the use of the field equations,

$$\begin{aligned}\delta S &= \int \{ \Pi^\mu \partial^2 \delta X^\mu + 2i \bar{\Psi}^\mu \rho^a \partial_a \delta \Psi^\mu \} d^2 \zeta = \int \{ \Pi^\mu \partial^2 (\bar{\epsilon} \Psi^\mu) + 2i \bar{\Psi}^\mu \rho^a \partial_a (-i \rho^b \partial_b \Pi^\mu \epsilon) \} d^2 \zeta \\ &= \int \{ \partial^2 \Pi^\mu (\bar{\epsilon} \Psi^\mu) + 2 \bar{\Psi}^\mu \rho^a \rho^b \partial_a \partial_b \Pi^\mu \epsilon \} d^2 \zeta = \int \{ \partial^2 \Pi^\mu \bar{\epsilon} \Psi^\mu - \bar{\Psi}^\mu \epsilon \partial^2 \Pi^\mu \} d^2 \zeta = 0\end{aligned}\quad (29)$$

We can use the Noether method to derive conserved supersymmetry current, the current is a spinor

$$J_a = \frac{1}{2} \rho^b \rho_a \Psi^\mu \partial_b \Pi^\mu \equiv (J_+, J_-), \quad (30)$$

here  $J_+$  and  $J_-$  are two component spinor currents. The current is conserved

$$\partial^a J_a = \frac{1}{2} \rho^b \rho_a \partial^a \Psi^\mu \partial_b \Pi^\mu + \frac{1}{2} \rho^b \rho_a \Psi^\mu \partial^a \partial_b \Pi^\mu = 0 \quad (31)$$

and

$$\rho^a J_a = \frac{1}{2} \rho^a \rho^b \rho_a \Psi^\mu \partial_b \Pi^\mu = 0 \quad (32)$$

because  $\rho^a \rho^b \rho_a = 0$ . In the two-dimensional notations the supercurrent has components:

$$J_+ = \Psi_+^\mu \partial_+ \Pi^\mu, \quad \partial_- J_+ = 0, \quad (33)$$

$$J_- = \Psi_-^\mu \partial_- \Pi^\mu, \quad \partial_+ J_- = 0, \quad (34)$$

representing the *generalized Dirac equations* in our case.

Let us summarize the symmetries of the system. They are: translation in the target space  $\delta_a$ , spacetime rotations  $\delta_\Lambda$ , fermi-bose transformation  $\delta_\epsilon$  and conformal transformations  $\delta_u$

$$\begin{aligned}\delta_a : \quad & \delta_a X^\mu = a^\mu & P_a^\mu &= \partial_a \Pi^\mu \\ & \delta_a \Psi^\mu = 0 \\ \delta_\Lambda : \quad & \delta_\Lambda X^\mu = \Lambda^{\mu\nu} X^\nu & M_a^{\mu\nu} &= X^\mu \partial_a \Pi^\nu - X^\nu \partial_a \Pi^\mu + \Pi^\mu \partial_a X^\nu - \Pi^\nu \partial_a X^\mu + i \Psi^\mu \rho_a \Psi^\nu \\ & \delta_\Lambda \Psi^\mu = \Lambda^{\mu\nu} \Psi^\nu \\ \delta_\epsilon : \quad & \delta_\epsilon X^\mu = \bar{\epsilon} \Psi^\mu & J_a &= \frac{1}{2} \rho^b \rho_a \Psi^\mu \partial_b \Pi^\mu \\ & \delta_\epsilon \Psi^\mu = -i \rho^a \partial_a \Pi^\mu \cdot \epsilon \\ \delta_u : \quad & \delta_u X^\mu = u^a \partial_a X^\mu & T_{ab} &= \frac{1}{2} \partial_{\{a} \Pi^\mu \partial_{b\}} X^\mu + \frac{i}{4} \bar{\Psi}^\mu \rho_{\{a} \partial_{b\}} \Psi^\mu - \text{trace} \\ & \delta_u \Psi^\mu = u^a \partial_a \Psi^\mu\end{aligned}\quad (35)$$

Now we can clearly see that two world sheet supersymmetry transformations are equivalent to translation  $\delta_a$  in (35)

$$\int_0^{2\pi} [\delta_1, \delta_2] X^\mu d\zeta^1 = 2i \bar{\epsilon}_1 \rho^a \epsilon_2 \int_0^{2\pi} \partial_a \Pi^\mu d\zeta^1 = 2i \bar{\epsilon}_1 \rho^0 \epsilon_2 \int_0^{2\pi} \partial_0 \Pi^\mu d\zeta^1 = 2i \bar{\epsilon}_1 \rho^0 \epsilon_2 P^\mu, \quad (36)$$

$$\int_0^{2\pi} [\delta_1, \delta_2] \Psi^\mu d\zeta^1 = 0. \quad (37)$$

Normally in standard string theory it is equivalent to a worldsheet conformal transformation.

Defining the conjugate variable for the fermion field as  $P_\psi^\mu = \frac{\delta S}{\delta \partial_0 \Psi^\mu} = \Psi^\mu$  we can get standard anticommutation relations

$$\{\Psi_\pm^\mu(\zeta), \Psi_\pm^\nu(\zeta')\} = \pi \eta^{\mu\nu} \delta(\zeta^1 - \zeta'), \quad (38)$$

with all others equal to zero. Making use of these commutators one can get the standard algebra of the two-dimensional conformal group (15), where now

$$\begin{aligned} T_{++} &= 2\partial_+ \Pi^\mu \partial_+ X^\mu + \frac{i}{2} \Psi_+^\mu \partial_+ \Psi_+^\mu, \\ T_{--} &= 2\partial_- \Pi^\mu \partial_- X^\mu + \frac{i}{2} \Psi_-^\mu \partial_- \Psi_-^\mu. \end{aligned} \quad (39)$$

We are able now to compute the anticommutator of two supercurrents:

$$\{J_+(\zeta), J_+(\zeta')\} = \{\Psi_+^\mu(\zeta), \Psi_+^\nu(\zeta')\} \cdot \partial_+ \Pi^\mu(\zeta) \partial_+ \Pi^\nu(\zeta') \equiv \pi P_+^2(\zeta) \delta(\zeta^1 - \zeta'). \quad (40)$$

The square of our generalized Dirac operator  $J_+ = P_+^\mu \Psi_+^\mu = \partial_+ \Pi^\mu \Psi_+^\mu$  is not any more equal to the Virasoro operator  $T_{++} = 2P_+^\mu \partial_+ X^\mu + (i/2) \Psi_+^\mu \partial_+ \Psi_+^\mu$ , but instead is a square of space-time translation operator  $P_+^\mu$ . One can also compute the following commutator

$$[T_{++}(\zeta), J_+(\zeta')] = i\pi J_+(\zeta) \delta'(\zeta - \zeta'). \quad (41)$$

Quantization of this theory is strait forward [6]. From the equivalent form of the action (9) we can deduce the propagator

$$\langle \Pi^\mu(k) X^\nu(-k) \rangle = \eta^{\mu\nu} \frac{i\pi}{k^2}, \quad (42)$$

or in the coordinate form

$$\langle \Pi^\mu(\zeta) X^\nu(\tilde{\zeta}) \rangle = -\frac{\eta^{\mu\nu}}{2} \ln(|\zeta - \tilde{\zeta}| \mu). \quad (43)$$

Using the fact that there is no correlations between right and left moving modes of the  $\Pi$  field and right and left moving modes of the  $X$  field we shall get

$$\langle \Pi_R^\mu(\zeta^-) X_R^\nu(\tilde{\zeta}^-) \rangle = -\eta^{\mu\nu} \ln[(\zeta^- - \tilde{\zeta}^-) \mu], \quad \langle \Pi_L^\mu(\zeta^+) X_L^\nu(\tilde{\zeta}^+) \rangle = -\eta^{\mu\nu} \ln[(\zeta^+ - \tilde{\zeta}^+) \mu] \quad (44)$$

Now we are in a position to compute the two point correlation function of the energy momentum operator for bosonic coordinates  $X, \Pi$

$$\begin{aligned} \langle T T_{++}^{boson}(\zeta^+) T_{++}^{boson}(\tilde{\zeta}^+) \rangle &= \frac{1}{4} \langle T : \dot{\Pi}_L^\mu(\zeta^+) \dot{X}_L^\mu(\zeta^+) :: \dot{\Pi}_L^\nu(\tilde{\zeta}^+) \dot{X}_L^\nu(\tilde{\zeta}^+) : \rangle \\ &= \frac{1}{4} \langle \dot{\Pi}_L^\mu(\zeta^+) \dot{X}_L^\nu(\tilde{\zeta}^+) \rangle \langle \dot{X}_L^\mu(\zeta^+) \dot{\Pi}_L^\nu(\tilde{\zeta}^+) \rangle = \frac{1}{4} \frac{D}{(\zeta^+ - \tilde{\zeta}^+)^4}. \end{aligned} \quad (45)$$

The contribution of the ghosts b-c system to the central charge remains the same as in the standard bosonic string theory

$$\langle T T_{++}^{ghost}(\zeta^+) T_{++}^{ghost}(\tilde{\zeta}^+) \rangle = -\frac{13}{4} \frac{1}{(\zeta^+ - \tilde{\zeta}^+)^4}.$$



The same is true for fermion and super-ghosts  $\gamma - \beta$  system

$$\begin{aligned} < T T_{++}^{fermion}(\zeta^+) T_{++}^{fermion}(\tilde{\zeta}^+) > + < T T_{++}^{super-ghost}(\zeta^+) T_{++}^{super-ghost}(\tilde{\zeta}^+) > = \\ = \frac{1}{16} \frac{D}{(\zeta^+ - \tilde{\zeta}^+)^4} + \frac{11}{8} \frac{1}{(\zeta^+ - \tilde{\zeta}^+)^4}. \end{aligned} \quad (46)$$

therefore the absence of conformal anomaly

$$\frac{D}{4} - \frac{13}{4} + \frac{D}{16} + \frac{11}{8} = 0 \quad (47)$$

requires that the space-time should be 6-dimensional <sup>6</sup>

$$D_c = 6. \quad (48)$$

This result can be qualitatively understood if one take into account the fact that the field equations here are of the forth order and therefore we have two time more degrees of freedom in the bosonic sector than in the standard string theory. It will be a subject of another paper to investigate the spectrum of this theory in full details.

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## References

- [1] G.K.Savvidy and K.G.Savvidy, Mod.Phys.Lett. A8 (1993) 2963  
G.K.Savvidy, JHEP 0009 (2000) 044  
R.V.Ambartzumian and et al, Phys.Lett. B275 (1992) 99  
G.K.Savvidy and K.G.Savvidy, Int.J.Mod.Phys. A8 (1993) 3993
- [2] A.M.Polyakov. Nucl.Phys.B268 (1986) 406  
H.Kleinert. Phys.Lett. 174B (1986) 335  
W.Helfrich. Z.Naturforsch. C28 (1973) 693; J.Phys.(Paris) 46 (1985) 1263  
L.Peliti and S.Leibler. Phys.Rev.Lett. 54 (1985) 1690  
D.Forster. Phys.Lett. 114A (1986) 115  
T.L.Curtright and et.al. Phys.Rev.Lett. 57 (1986)799; Phys.Rev. D34 (1986) 3811  
F.David. Europhys.Lett. 2 (1986) 577  
P.O.Mazur and V.P.Nair. Nucl.Phys. B284 (1987) 146  
E.Braaten and C.K.Zachos. Pys.Rev. D35 (1987) 1512  
E.Braaten, R.D.Pisarski and S.M.Tye. Phys.Rev.Lett. 58 (1987) 93  
P.Olesen and S.K.Yang. Nucl.Phys. B283 (1987) 73  
R.D.Pisarski. Phys.Rev.Lett. 58 (1987) 1300
- [3] D.Weingarten. Nucl.Phys.B210 (1982) 229  
T.Sterling and J.Greensite. Phys.Lett. B121 (1983) 345  
B.Durhuus,J.Fröhlich and T.Jonsson. Nucl.Phys.B225 (1983) 183

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<sup>6</sup>We got the same result using mode expansion, the details will be given elsewhere.

- J.Ambjørn,B.Durhuus,J.Fröhlich and T.Jonsson. Nucl.Phys.B290 (1987) 480  
T.Hofsäss and H.Kleinert. Phys.Lett. A102 (1984) 420  
M.Karowski and H.J.Thun. Phys.Rev.Lett. 54 (1985) 2556  
F.David. Europhys.Lett. 9 (1989) 575
- [4] R. Manvelian and G. Savvidy, Phys.Lett.B533 (2002) 138
  - [5] T.Curtright and P. van Nieuwenhuizen, Nucl.Phys.B294 (1987)125
  - [6] G.K. Savvidy, Conformal invariant strings with extrinsic curvature, hep-th 0202108
  - [7] R. Gopakumar, S. Minwalla, N. Seiberg and A. Strominger, JHEP **0008**, 008 (2000)
  - [8] P.Ramond, Phys.Rev.D3 (1971) 2415
  - [9] A.Neveu and J.Schwarz, Nucl.Phys.B31 (1971) 86
  - [10] J.L.Gervais and B.Sakita, Nucl.Phys.B34 (1971) 632  
Y.Iwasaki and K.Kikkawa,Phys.Rev.D8 (1973) 440  
B.Zumino, "Relativistic Strings and Supergauges" pp.367-381 in "Renormalisation and Invariance in QFT" ed. E.Caianiello (Plenum Press, 1974)  
L.Brink,P. Di Vecchia and P.Howe, Phys.Lett.65B (1976) 471  
S.Deser and B.Zumino, Phys.Lett.65B (1976) 369  
A.M.Polyakov. Phys.Lett.103B (1981) 207; Phys.Lett.103B (1981) 211
  - [11] M.B.Green, J.H.Schwarz and E.Witten, Superstring Theory. Vol.1,2 Cambridge: Cambridge University Press (1997).
  - [12] J. Polchinski, String Theory. Vol.1,2 Cambridge: Cambridge University Press (1998)